Homework Assignment No. 2 - Solutions

Problem 1 - (10 points)

a) For the emitter follower output stage shown below, find the value of R_1 for maximum efficiency and find the value of that efficiency. $V_{CC}=-V_{EE}=2.5V$, $V_{CE}(sat)=0.2V$, $R_L=10k\Omega$, $V_{BE}(on)=0.7V$.

b) The load resistance R_L is replaced with a capacitor of 100pF. If the input voltage suddenly drops from 2.5V to -2.5V, explain what happens at the output and accurately sketch the output voltage as a function of time, specifying its initial and final values and time interval.

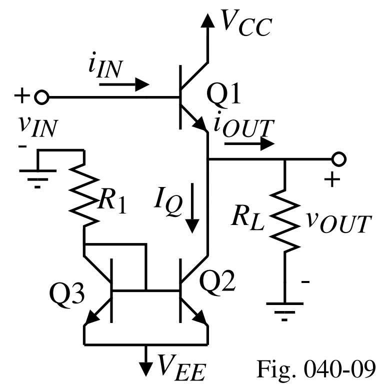


Fig. 040-09

Solution

The I_Q for maximum efficiency is found as,

$$I_Q = \left(\frac{V_{CC} - V_{CE}(sat)}{R_L}\right) = 230\mu A$$

$$R_1 = \left(\frac{-V_{EE} - V_{BE}}{I_Q}\right) = 7.826k\Omega$$

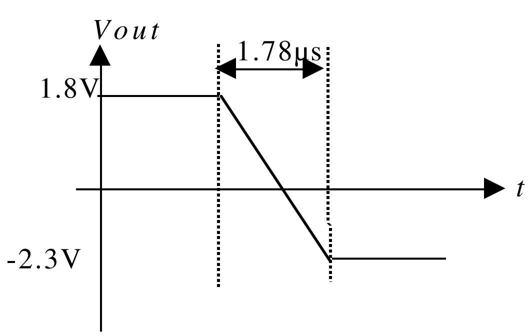
$$P_L(\max) = \left(\frac{V_{CC} - V_{CE}(sat)}{\sqrt{2}}\right)\left(\frac{I_Q}{\sqrt{2}}\right) = 0.5(2.3V)(0.23mA) = 0.2645mW$$

$$P_{supply} = 2V_{CC}I_Q = 2(2.5)(0.23mA) = 1.15mW$$

$$\eta = \frac{P_{L(\max)}}{P_{\sup ply}} = \frac{1}{4}\left(1 - \frac{V_{CE(sat)}}{V_{CC}}\right) = 23\%$$

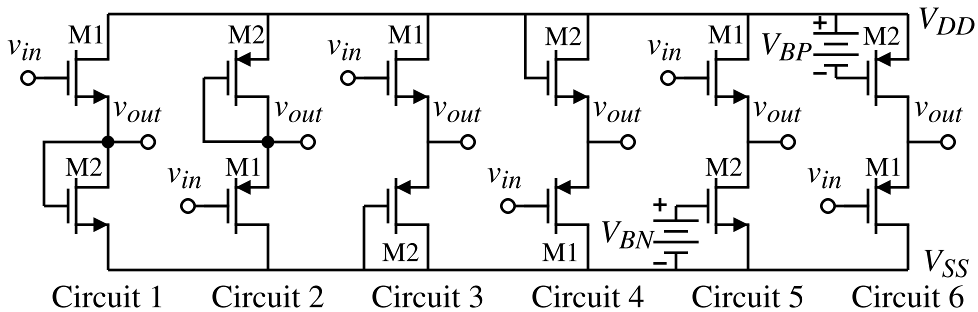
b) The output would slew under such condition. The current will be limited by the bias current:

Slew rate=0.23mA/100pF=2.3V/μs



Problem 2 - (10 points)

Six versions of a source follower are shown below. Assume that $K'_N = 2K'_P$, $\lambda_P = 2\lambda_N$, all W/L ratios of all devices are equal, and that all bias currents in each device are equal. Neglect bulk effects in this problem and assume no external load resistor. Identify which circuit or circuits have the following characteristics: (a.) highest small-signal voltage gain, (b.) lowest small-signal voltage gain, (c.) the highest output resistance, (d.) the lowest output resistance, (e.) the highest v_{out}(max) and (f.) the lowest v_{out}(max).



FS02E1P1

Solution

(a.) and (b.) - Voltage gain. Small signal model:

The voltage gain is found as: $\frac{v_{out}}{v_{in}} = \frac{g_m}{g_m+G_L}$

where G_L is the load conductance. Therefore we get:

Circuit	1	2	3	4	5	6
$\frac{v_{out}}{v_{in}}$	$\frac{g_{mN}}{g_{mN}+g_{mN}}$	$\frac{g_{mP}}{g_{mP}+g_{mP}}$	$\frac{g_{mN}}{g_{mN}+g_{mP}}$	$\frac{g_{mP}}{g_{mP}+g_{mN}}$	$\frac{g_{mN}}{g_{mN}+g_{dsN}+g_{dsP}}$	$\frac{g_{mP}}{g_{mP}+g_{dsN}+g_{dsP}}$

But $g_{mN} = \sqrt{2}\, g_{mP}$ and $g_{dsN} = 0.5 g_{dsP}$, therefore

Circuit	1	2	3	4	5	6
$\frac{v_{out}}{v_{in}}$	$\frac{1}{2}$	$\frac{1}{2}$	0.5858	0.4142	$\frac{g_{mP}}{g_{mP}+(g_{dsP}+g_{dsN})/\sqrt{2}}$	$\frac{g_{mP}}{g_{mP}+g_{dsP}+g_{dsN}}$

Thus, circuit 5 has the highest gain and circuit 4 the lowest gain

(c.) and (d.) - Output resistance.

The denominators of the first table show the following:

Ckt.6 has the highest output resistance and Ckt. 1 the lowest output resistance.

(e.) Assuming no current has to be provided by the output, circuits 2, 4, and 6 can pull the output to V_{DD}. $\therefore$ Circuits 2, 4 and 6 have the highest output swing.

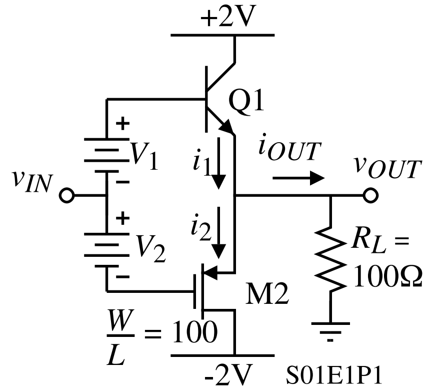
(f.) Assuming no current has to be provided by the output, circuits 1, 3, and 5 can pull the output to ground. $\therefore$ Circuits 1, 3 and 5 have lowest output swing.

Summary

(a.) Ckt. 5 has the highest voltage gain
(b.) Ckt. 4 has the lowest voltage gain
(c.) Ckt. 6 has the highest output resistance
(d.) Ckt. 1 has the lowest output resistance
(e.) Ckts. 2,4 and 6 have the highest output
(f.) Ckts. 1,3 and 5 have the lowest output

Problem 3 - (10 points)

A push-pull follower is shown which uses an NPN BJT and a p-channel MOSFET. In this problem, ignore the bulk effect, the channel length modulation, and the Early voltage. The parameters for the NPN BJT are β_F = 100, I_s = 10fA and V_t = 25.9mV. The model parameters for the PMOS are K_P' = 50μA/V^2 and V_T = -0.7V. (a.) Find the value of the dc batteries, V_1 and V_2, which will cause 100μA to flow in Q1 and M2 when the dc value of v_{IN} = 0VDC. (b.) Find the small-signal input resistance, output resistance and voltage gain when the dc value of v_{IN} = 0VDC.



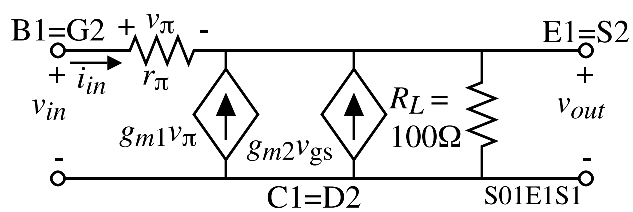
S01E1P1

Solution

(a.) $V_1 = V_{BE1} = V_t \ln\left(\frac{i_C}{I_s}\right) = 0.0259\ln\left(\frac{100\mu A}{10fA}\right) = 0.5964V$ _ $\boxed{V_1 = 0.5964V}$

$V_2 = V_{SG2} = \sqrt{\frac{2I_D}{K_P'(W/L)}} + |V_{TP}| = \sqrt{\frac{2\cdot 100}{50\cdot 100}} + 0.7 = 0.9V$ __ $\boxed{V_2 = 0.9V}$

(b.) Small-signal model (simplified):



S01E1S1

$g_{m1} = \frac{I_{C1}}{V_t} = \frac{100\mu A}{25.9mV} = 3.86mS$

$r_{\pi 1} = \frac{1+\beta_F}{g_{m1}} = 26.159k\Omega$

$g_{m2} = \sqrt{\frac{2K_P' W_2 I_{D2}}{L_2}} = \sqrt{2\cdot 50\cdot 100\cdot 100} = 1mS$

R_{in}: $v_{in} = r_{\pi 1}i_{in} + (i_{in} + g_{m1}v_\pi + g_{m2}\, v_{gs2})R_L = r_{\pi 1}i_{in} + (i_{in} + g_{m1}r_{\pi 1}i_{in} + g_{m2}\, r_{\pi 1}i_{in})R_L$

$R_{in} = \frac{v_{in}}{i_{in}} = r_{\pi 1} + R_L + g_{m1}r_{\pi 1}R_L + g_{m2}\, r_{\pi 1}R_L = r_{\pi 1} + R_L(1+\beta_F) + g_{m2}\, r_{\pi 1}R_L$

$\therefore R_{in} = 26.159k\Omega + 101\cdot 100\Omega + 1\cdot 26.159k\Omega \cdot 0.1 = 38.875k\Omega$ $\boxed{R_{in} = 38.875k\Omega}$

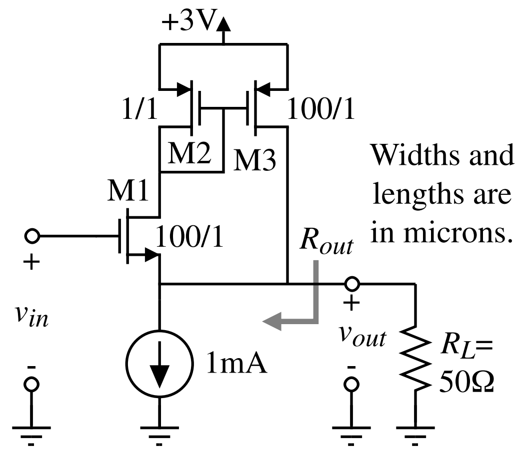
R_{out}: $R_{out} = \frac{1}{g_{m1}} \| \frac{1}{g_{m2}} = \frac{1}{3.86mS+1mS} = 205.8k$_ __ $\boxed{R_{out} = 205.8kW}$

$\frac{v_{out}}{v_{in}}$: $\frac{v_{out}}{v_{in}} = \frac{v_{out}}{i_{in}}\frac{i_{in}}{v_{in}} = \frac{R_L(1+b_F)+g_{m2}r_{p1}R_L}{r_{p1}+R_L(1+b_F)+g_{m2}r_{p1}R_L} = \frac{12.716}{38.875} = 0.3271$

$\boxed{\frac{v_{out}}{v_{in}} = 0.3271V/V}$

Problem 4 - (10 points)

Find an algebraic expression for the voltage gain, v_{out}/v_{in}, and the output resistance, R_{out}, of the source follower shown in terms of the small-signal model parameters, g_m and R_L (ignore r_{ds}). If the bias current is 1mA find the numerical value of the voltage gain and the output resistance. Assume that $K_N' = 110\mu A/V^2$, $V_{TN} = 0.7V$, and $K_P' = 50\mu A/V^2$, $V_{TP} = -0.7V$.



S03E1P1

Solution

A small-signal model for this circuit is shown below neglecting r_{ds} of the transistors.

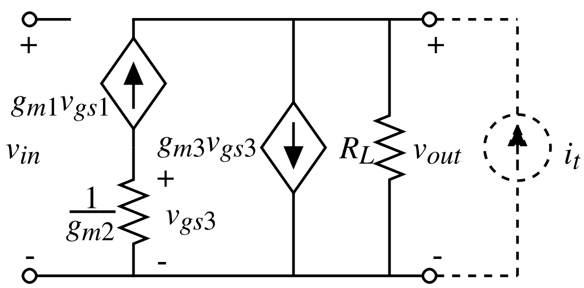


Fig. S03E1S1

Summing currents at the output node gives,

$$g_{m1}v_{gs1} = g_{m3}v_{gs3} + G_L v_{out}$$

Also, $v_{gs3} = -\, g_{m1}v_{gs1}(1/g_{m2})$

$$\therefore \quad g_{m1}v_{gs1} = g_{m3}\left(-\frac{g_{m1}}{g_{m2}}\right)v_{gs1} + G_L v_{out}$$

$$g_{m1}v_{gs1}\left(1 + \frac{g_{m3}}{g_{m2}}\right) = G_L v_{out} \rightarrow$$

$$g_{m1}(v_{out} - v_{in})\left(1 + \frac{g_{m3}}{g_{m2}}\right) = G_L v_{out}$$

$$\therefore \quad \boxed{\frac{v_{out}}{v_{in}} = \frac{g_{m1}\left(1+\frac{g_{m3}}{g_{m2}}\right)}{g_{m1}\left(1+\frac{g_{m3}}{g_{m2}}\right) + G_L}}$$

Setting $v_{in} = 0$ and applying i_t and solving for v_{out} and ignoring R_L gives,

$$i_t = g_{m3}v_{gs3} + g_{m1}v_{out} = g_{m3}\left(\frac{g_{m1}}{g_{m2}}\right)v_{out} + g_{m1}v_{out}$$

$$\therefore \quad \frac{v_{out}}{i_t} = \boxed{R_{out} = \frac{1}{g_{m1}\left(1+\frac{g_{m3}}{g_{m2}}\right)}}$$

Note that the 1mA splits between M1(M2) and M3 in a ratio of 1 to 100. Therefore, $I_{D1} = I_{D2} = 9.9\mu A$ and $I_{D3} = 990.1\mu A$.

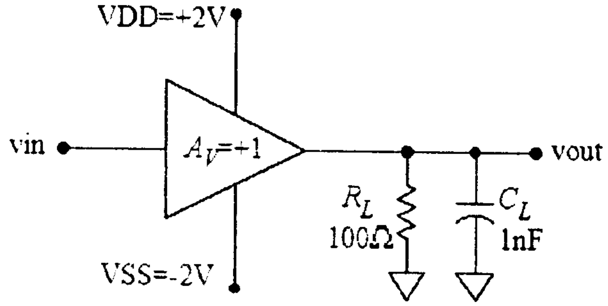
$\therefore \; g_{m1} = \sqrt{2\cdot 110\cdot 100\cdot 9.9} = 466.71\mu S$, $g_{m2} = \sqrt{2\cdot 50\cdot 1\cdot 9.9} = 31.47\mu S$

and $g_{m3} = \sqrt{2\cdot 110\cdot 100\cdot 990.1} = 3146.7\mu S$

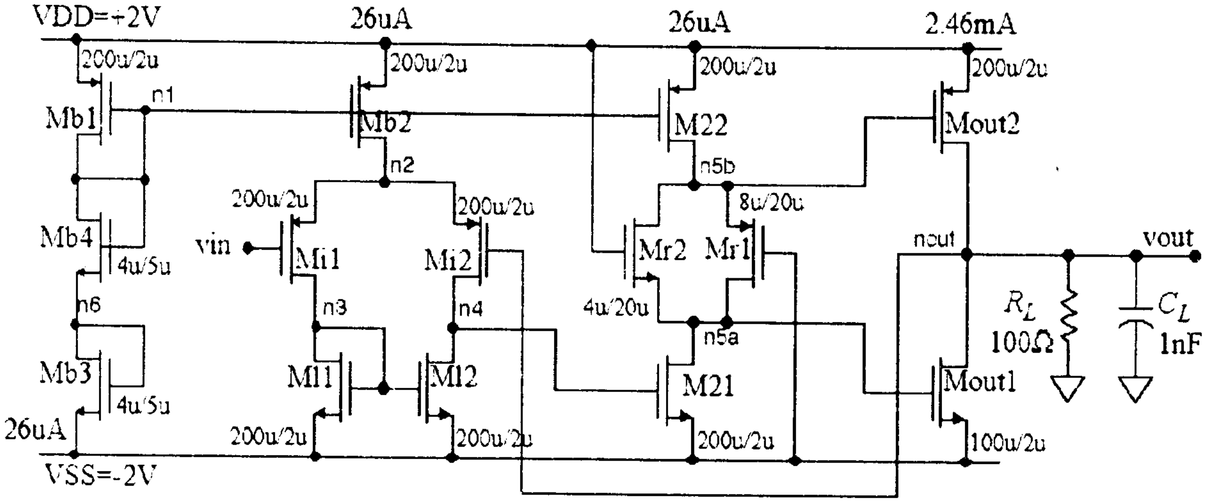
$$\frac{v_{out}}{v_{in}} = \frac{466.71\cdot 101}{466.71\cdot 101 + 1/50} = \frac{47.137}{47.137 + 20} = \underline{\underline{0.702 \text{ V/V}}}$$

$$R_{out} = \frac{1000}{47.137} = \underline{\underline{21.2\Omega}}$$

Problem5 - Design Prob.#1



Output swing	-1.25V, 1.06V
SR	> 10V/usec
DC gain	= 1
efficiency	= 30.6%
score	40

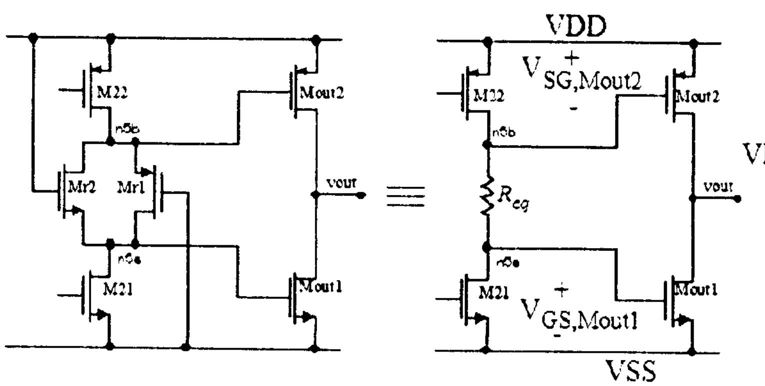


A three stage amplifier connected in unity gain configuration is used as the output buffer. The first two stages are class-A whereas the last stage is class-AB.

1nF load capacitance makes the pole present at node nout the dominant pole. Therefore, there is no need to apply additional compansation to the three stage amplifier.

DC gain of the amplifier in unity gain configuration = $A_v/(1+A_v)$ where $A_v \propto g_m^3 r_{ds}^3$.

Output impedance of the unity gain buffer $= r_{ds}/(1+A_v)$.



$$VDD - VSS = V_{GS,Mout1} + V_{SG,Mout2} + I_{d,M21} Req$$

$I_{d,M21}$: quiescent drain current of M21.

Mr1 and Mr2 act as a linear resistor, allowing a voltage drop between n5a and n5b. The net result is reduced quiescent current at the output stage.

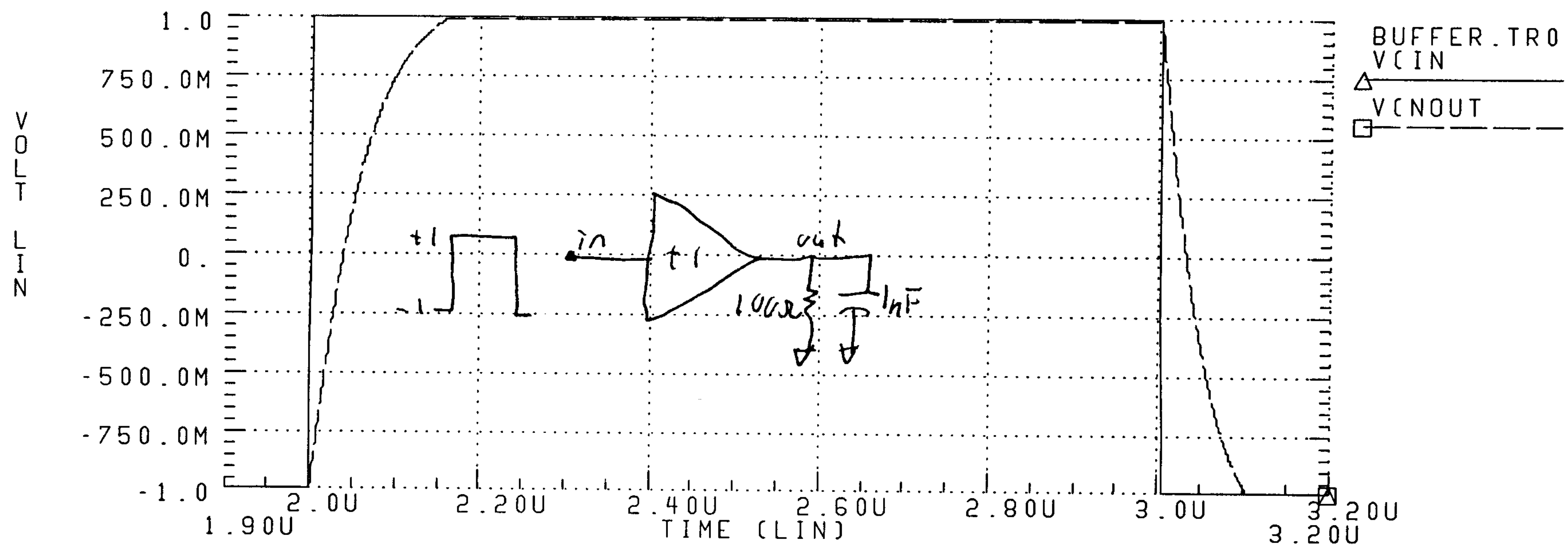
```
*output buffer
.option brief
.options
+ post
+ ingold=2
+ scale=1e-6
+ accurate
+ delmax=0.5n
+ method=gear lvltim=2
+ probe
*

vdd dd 0 dc  2v
vss ss 0 dc -2v
*vin in 0 dc 0v
vin in 0 sin (0 1.5 100k 0)
*vin in 0 pulse (-1 1 2u 2n 2n 1u 2u)

mb1 n1 n1 dd dd cmosp l=2u w=200u
mb2 n2 n1 dd dd cmosp l=2u w=200u
mb3 n6 n6 ss ss cmosn l=5u w=4u
mb4 n1 n1 n6 ss cmosn l=5u w=4u
mi1 n3 in n2 n2 cmosp l=2u w=200u
mi2 n4 nout n2 n2 cmosp l=2u w=200u
ml1 n3 n3 ss ss cmosn l=2u w=200u
ml2 n4 n3 ss ss cmosn l=2u w=200u
m21 n5a n4 ss ss cmosn l=2u w=200u
m22 n5b n1 dd dd cmosp l=2u w=200u
mr1 n5a ss n5b dd cmosp l=20u w=8u
mr2 n5b dd n5a ss cmosn l=20u w=4u
mout1 nout n5a ss ss cmosn l=2u w=100u
mout2 nout n5b dd dd cmosp l=2u w=200u
rl nout 0 100
cl nout ss 1nF
*.dc vin -2 2 1m
.meas tran current_vdd avg i(vdd) from=0 to=50u     ] calculates the average
.meas tran power_vdd param='4*current_vdd'          ] current drawn from VDD.
                                                       and power

.tran 1u 50u
.model cmosn nmos kp=110u vto=0.7 lambda=0.01 gamma=0.4 phi=0.7
.model cmosp pmos kp=50u vto=-0.7 lambda=0.01 gamma=0.7 phi=0.8
.probe v(in) v(n1) v(n2) v(n3) v(n4) v(n5) v(nout) v(n5a) v(n5b) i(vdd) i(vss)
.op
.end
```

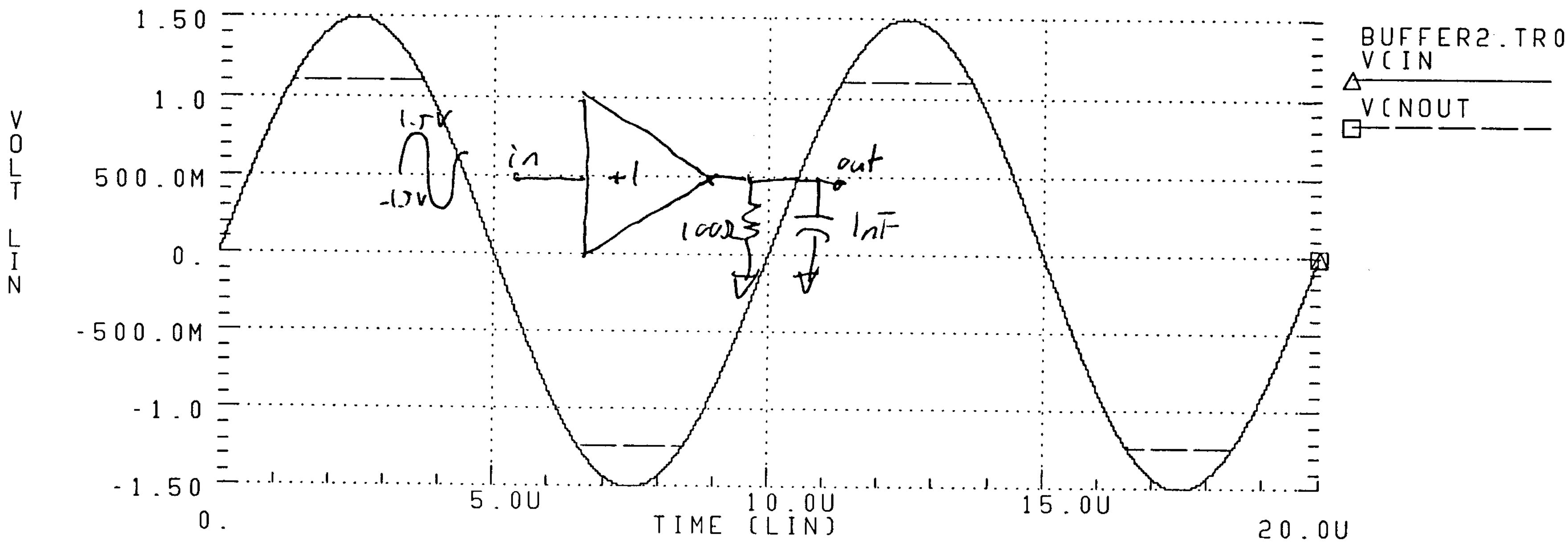
```
 ******  operating point information      tnom=  25.000 temp=  25.000
******
***** operating point status is all       simulation time is     0.
   node     =voltage       node    =voltage       node    =voltage

+0:dd       = 2.000e+00 0:in      =    0.       0:n1      = 1.197e+00
+0:n2       = 7.723e-01 0:n3      =-1.250e+00 0:n4        =-1.230e+00
+0:n5a      =-3.641e-01 0:n5b     = 3.185e-01 0:n6        =-5.283e-01
+0:nout     =-1.441e-05 0:ss      =-2.000e+00

****  voltage sources

subckt
element  0:vdd       0:vss      0:vin
 volts    2.000e+00 -2.000e+00    0.
 current -2.537e-03  2.537e-03    0.
 power    5.073e-03  5.074e-03    0.

    total voltage source power dissipation=  1.015e-02       watts

**** resistors

subckt
element  0:rl
 r value  1.000e+02
 v drop  -1.441e-05
 current -1.441e-07
 power    2.075e-12

**** mosfets

subckt
element  0:mb1      0:mb2      0:mb3      0:mb4      0:mi1      0:mi2
model    0:cmosp    0:cmosp    0:cmosn    0:cmosn    0:cmosp    0:cmosp
 id      -2.659e-05 -2.670e-05  2.659e-05  2.659e-05 -1.335e-05 -1.335e-05
 ibs        0.         0.         0.     -1.472e-14    0.         0.
 ibd      8.027e-15  1.228e-14 -1.472e-14 -3.197e-14  2.023e-14  2.003e-14
 vgs     -8.027e-01 -8.027e-01  1.471e+00  1.725e+00 -7.723e-01 -7.724e-01
 vds     -8.027e-01 -1.227e+00  1.471e+00  1.725e+00 -2.023e+00 -2.003e+00
 vbs        0.         0.         0.     -1.471e+00    0.         0.
 vth     -7.000e-01 -7.000e-01  7.000e-01  9.548e-01 -7.000e-01 -7.000e-01
 vdsat   -1.027e-01 -1.027e-01  7.717e-01  7.708e-01 -7.235e-02 -7.236e-02
 beta     5.040e-03  5.061e-03  8.930e-05  8.952e-05  5.101e-03  5.100e-03
 gam eff  7.000e-01  7.000e-01  4.000e-01  4.000e-01  7.000e-01  7.000e-01
 gm       5.177e-04  5.199e-04  6.891e-05  6.900e-05  3.690e-04  3.690e-04
 gds      2.638e-07  2.638e-07  2.620e-07  2.614e-07  1.308e-07  1.309e-07
 gmb      2.026e-04  2.034e-04  1.647e-05  9.364e-06  1.444e-04  1.444e-04
 cdtot    1.478e-28  2.261e-28    0.         0.      3.726e-28  3.689e-28
 cgtot    9.856e-26  9.864e-26  4.651e-27  4.641e-27  1.009e-25  1.009e-25
 cstot    9.208e-26  9.208e-26  4.604e-27  4.604e-27  9.208e-26  9.208e-26
 cbtot    6.331e-27  6.331e-27    0.         0.      8.414e-27  8.412e-27
 cgs      9.208e-26  9.208e-26  4.604e-27  4.604e-27  9.208e-26  9.208e-26
 cgd      1.478e-28  2.261e-28    0.         0.      3.726e-28  3.689e-28

subckt
element  0:ml1      0:ml2      0:m21      0:m22      0:mr1      0:mr2
model    0:cmosn    0:cmosn    0:cmosn    0:cmosp    0:cmosp    0:cmosn
 id       1.335e-05  1.335e-05  2.682e-05 -2.682e-05  1.100e-05  1.582e-05
 ibs        0.         0.         0.         0.      2.364e-14 -1.636e-14
 ibd     -7.491e-15 -7.693e-15 -1.636e-14  1.682e-14  1.682e-14 -2.318e-14
```

```
 vgs       7.491e-01  7.491e-01  7.693e-01 -8.027e-01 -1.635e+00  2.364e+00
 vds       7.491e-01  7.693e-01  1.635e+00 -1.681e+00  6.826e-01  6.826e-01
 vbs         0.         0.         0.         0.        2.364e+00 -1.635e+00
 vth       7.000e-01  7.000e-01  7.000e-01 -7.000e-01 -1.176e+00  9.767e-01
 vdsat     4.908e-02  4.908e-02  6.927e-02 -1.027e-01 -6.826e-01  6.826e-01
 beta      1.108e-02  1.108e-02  1.118e-02  5.084e-03  2.014e-05  2.215e-05
 gam eff   4.000e-01  4.000e-01  4.000e-01  7.000e-01  7.000e-01  4.000e-01
 gm        5.440e-04  5.441e-04  7.744e-04  5.222e-04  1.375e-05  1.512e-05
 gds       1.325e-07  1.325e-07  2.639e-07  2.638e-07  9.357e-06  1.577e-05
 gmb       1.300e-04  1.301e-04  1.851e-04  2.044e-04  3.054e-06  1.979e-06
 cdtot     1.380e-28  1.417e-28  3.013e-28  3.097e-28  3.380e-26  1.032e-26
 cgtot     9.994e-26  9.994e-26  9.836e-26  9.873e-26  5.208e-26  2.669e-26
 cstot     9.208e-26  9.208e-26  9.208e-26  9.208e-26  1.810e-26  1.633e-26
 cbtot     7.715e-27  7.715e-27  5.970e-27  6.331e-27  1.729e-28    0.
 cgs       9.208e-26  9.208e-26  9.208e-26  9.208e-26  1.810e-26  1.633e-26
 cgd       1.380e-28  1.417e-28  3.013e-28  3.097e-28  3.380e-26  1.032e-26

subckt
element   0:mout1    0:mout2
model     0:cmosn    0:cmosp
 id        2.457e-03 -2.457e-03
 ibs         0.         0.
 ibd      -2.000e-14  2.000e-14
 vgs       1.635e+00 -1.681e+00
 vds       2.000e+00 -2.000e+00
 vbs         0.         0.
 vth       7.000e-01 -7.000e-01
 vdsat     9.359e-01 -9.815e-01
 beta      5.610e-03  5.100e-03
 gam eff   4.000e-01  7.000e-01
 gm        5.250e-03  5.006e-03
 gds       2.409e-05  2.408e-05
 gmb       1.255e-03  1.959e-03
 cdtot     1.842e-28  3.683e-28
 cgtot     4.651e-26  9.323e-26
 cstot     4.604e-26  9.208e-26
 cbtot     2.788e-28  7.758e-28
 cgs       4.604e-26  9.208e-26
 cgd       1.842e-28  3.683e-28

 Opening plot unit= 15
file=./buffer.tr0

******
*current source
 ******  transient analysis                 tnom=  25.000 temp=  25.000
******
  current_vdd      = -4.5829E-03  from=   .0000E+00     to=  5.0000E-05
  power_vdd        = -1.8332E-02  average power dissipation

        ***** job concluded
******  Star-HSPICE  --    97.2.1 (970915) 14:15:30  98/06/01    pa
******
*current source
 ******  job statistics summary             tnom=  25.000 temp=  25.000
******

         total memory used        159 kbytes

 # nodes =     12 # elements=     19
```

```
  # diodes=      0 # bjts    =      0 # jfets   =      0 # mosfets =    14

     analysis      time      # points  tot. iter  conv.iter

     op point           .11         1          7
     transient       214.78        51     200387    100051 rev=    27
     readin             .10
     errchk             .08
     setup              .00
     output            1.55
           total cpu time        216.72 seconds
               job started at  14:15:30  98/06/01
               job ended   at  14:19:15  98/06/01


 lic: Release  token(s)
HSPICE job buffer.sp completed.
Mon Jun  1 14:19:15 PDT 1998
```

OUTPUT BUFFER





OUTPUT BUFFER