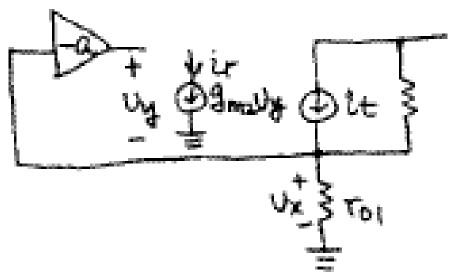
Homework Assignment No. 11 - Solutions

Problem 1 – (10 points)

Problem 8.26 of GHLM

(a) g_{m2} is the controlled source.



$$R_{out}(g_{m2}=0) = r_{o1}+r_{o2}$$

$$R(\text{short}) = (r_{o1}\|r_{o2})(a+1)g_{m2}$$

$$R(\text{open}) = 0$$

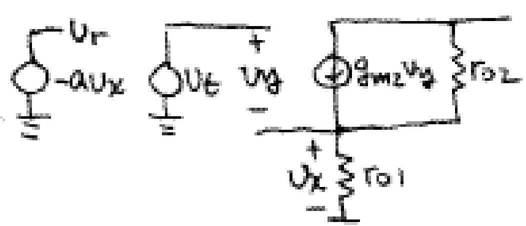
$$R_{out} = R_{out}(g_{m2}=0)\frac{1+R(\text{short})}{1+R(\text{open})}$$

$$= (r_{o1}+r_{o2})\frac{1+(r_{o1}\|r_{o2})(a+1)g_{m2}}{1+0}$$

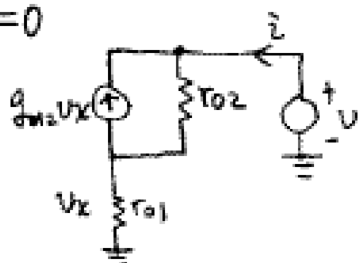
$$= r_{o1}+r_{o2}+(a+1)g_{m2}r_{o1}r_{o2}$$

$$\approx a g_{m2}r_{o1}r_{o2}$$

(b) a is the controlled source.



$a=0$



$$\frac{v_x}{r_{o1}} = g_{m2}v_x + \frac{v_x - v}{r_{o2}}$$

$$R_{out}(a=0) = \frac{v}{i} = \frac{v}{v_x/r_{o1}}$$

$$= g_{m2}r_{o1}r_{o2} + r_{o1} - r_{o2}$$

$$\approx g_{m2}r_{o1}r_{o2}$$

The output is short

$$v_x = g_{m2}(v_t - v_x)(r_{o1}\|r_{o2})$$

$$v_x = \frac{g_{m2}(r_{o1}\|r_{o2})}{1+g_{m2}(r_{o1}\|r_{o2})}v_t$$

$$R(\text{short}) = a\frac{g_{m2}(r_{o1}\|r_{o2})}{1+g_{m2}(r_{o1}\|r_{o2})}$$

$R(\text{open}) = 0$ ($v_x=0$ when the output is open)

$$R_{out} = R_{out}(a=0)\frac{1+R(\text{short})}{1+R(\text{open})}$$

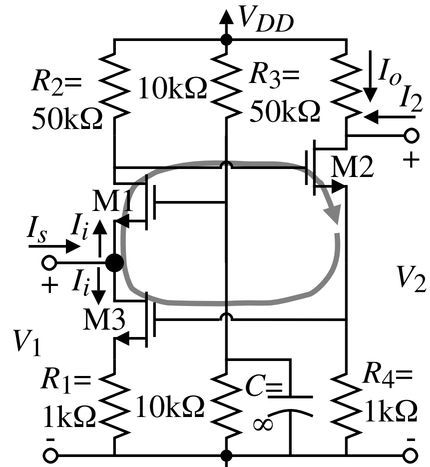
$$\approx g_{m2}r_{o1}r_{o2}\frac{1+a\frac{g_{m2}(r_{o1}\|r_{o2})}{1+g_{m2}(r_{o1}\|r_{o2})}}{1+0}$$

$$\approx a g_{m2}r_{o1}r_{o2}$$

(c) The results are the same, as they should be, even though the terms $R_{out}(k=0)$, $R(\text{open})$, and $R(\text{short})$ differ in (a) and (b).

Problem 2 - (10 points)

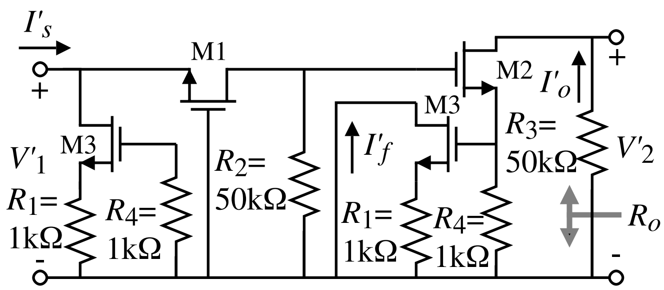
The simplified schematic of a feedback amplifier is shown. Use the method of feedback analysis to find V_2/V_1, $R_{in} = V_1/I_1$, and $R_{out} = V_2/I_2$. Assume that all transistors are matched and that g_m = 1mA/V and $r_{ds} = \infty$.



S02E3S1A

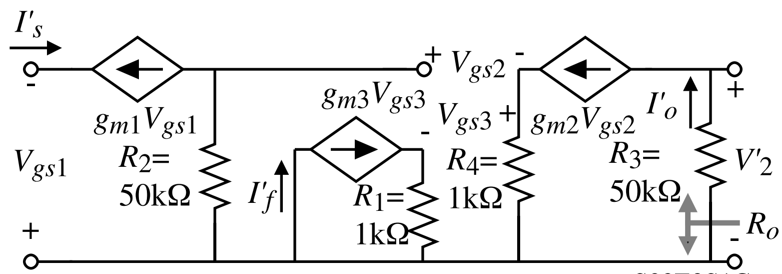
Solution

This feedback circuit is shunt-series. We begin with the open-loop, quasi-ac model shown below.



S02E3S1B

The small-signal, open-loop model is:



S02E3S1C

$$\frac{I'_o}{I'_s} = \left(\frac{I'_o}{V_{gs2}}\right)\left(\frac{V_{gs2}}{V_{gs1}}\right)\left(\frac{V_{gs1}}{I'_s}\right)$$

$V_{gs2} =$

$-g_{m1}V_{gs1}R_2 - g_{m2}V_{gs2}R_4$

or

$$\frac{V_{gs2}}{V_{gs1}} = \frac{-g_{m1}R_2}{1 + g_{m2}R_4} = -\frac{50}{2} = -25 \qquad \therefore\ a = \frac{I'_o}{I'_s} = (g_{m2})(-25)\left(\frac{-1}{g_{m1}}\right) = 25\text{A/A}$$

$$f = \frac{I'_f}{I'_o} = \left(\frac{I'_f}{V_{gs3}}\right)\left(\frac{V_{gs3}}{I'_o}\right) = (g_{m3})\left(\frac{R_4}{1 + g_{m3}R_1}\right) = (1\text{mA/V})(0.5\text{k}\Omega) = 0.5$$

$\therefore\ af = 25\cdot 0.5 = 12.5$

$$R_i = \frac{v'_1}{I'_s} = \frac{1}{g_{m1}} = 1\text{k}\Omega \ \rightarrow\ \boxed{R_{in} = R_{if} = \frac{R_i}{1 + af} = \frac{1000}{13.5} = 74.07\Omega}$$

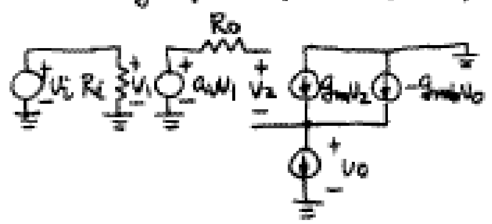
$$\boxed{R_{out} = 50\text{k}\Omega \ \ (R_3 \text{ is outside the feedback loop})}$$

$$\frac{I_o}{I_s} = \frac{a}{1+af} = \frac{25}{1+12.5} = 1.852\text{ A/A} \ \rightarrow\ \boxed{\frac{v_2}{v_1} = \frac{I_o(-50\text{k}\Omega)}{I_s(74.07\Omega)} = -1240.1\text{ V/V}}$$

Problem 3 – (10 points)

(a)

The basic amplifier without the feedback signal inserted at the inverting input of the opamp



$g_m = \sqrt{2K'\frac{W}{L}I_D} = \sqrt{2\times 80\times 10^{-6}\times 100\times 0.5\times 10^{-3}}$
$= 4.2\times 10^{-3}\ A/V$

$g_{mb} = \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}} g_m = \frac{\gamma}{2\sqrt{2\phi_f}} g_m$
$= \frac{0.3}{2\sqrt{2\times 0.3}} 4.2\times 10^{-3} = 8.1\times 10^{-4}\ A/V$

$v_o = g_m(a_v v_i - v_o)\frac{1}{g_{mb}}$

$a = \frac{v_o}{v_i} = a_v \frac{g_m}{g_m + g_{mb}}$

$f = 1$

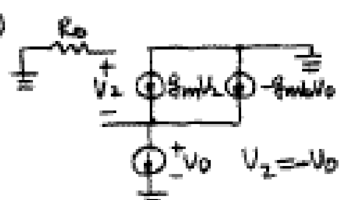
$af = a_v \frac{g_m}{g_m + g_{mb}} = 1000\frac{4.2}{4.2 + 0.81} = 838$

$A = \frac{a}{1+af} = \frac{838}{1+838} = 0.999$

$r_{ia} = R_i$

$R_{in} = r_{ia}(1+af) = R_i(1+af)$
$= 1M(1+838) = 839\ M\Omega$

$v_i = 0$



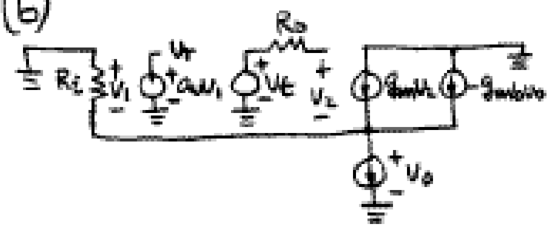
$v_2 = -v_o$

$r_{oa} = \frac{1}{g_m} \| \frac{1}{g_{mb}} = \frac{1}{g_m + g_{mb}}$

$R_{out} = \frac{r_{oa}}{1+af} = \frac{1}{g_m + g_{mb}}\frac{1}{1+af}$

$= \frac{1}{4.2\times 10^{-3} + 8.1\times 10^{-4}}\frac{1}{1+838}$
$= 0.238\,\Omega$

(b)



$v_o = g_m(v_t - v_o)(\frac{1}{g_{mb}} \| R_i)$

$v_o = \frac{g_m}{\frac{1}{R_i} + g_m + g_{mb}}$

$R = a_v \frac{g_m}{\frac{1}{R_i} + g_m + g_{mb}}$
$\approx a_v \frac{g_m}{g_m + g_{mb}}$
$= 838$

$A_\infty = \frac{v_o}{v_i}\Big|_{a_v=\infty} = 1 \quad (v_1 = 0 \text{ and } v_o = v_i)$

$d = \frac{v_o}{v_i}\Big|_{a_v=0} = \frac{\frac{1}{g_m} \| \frac{1}{g_{mb}}}{R_i + \frac{1}{g_m} \| \frac{1}{g_{mb}}}$

$= \frac{\frac{1}{g_m + g_{mb}}}{R_i + \frac{1}{g_m + g_{mb}}}$

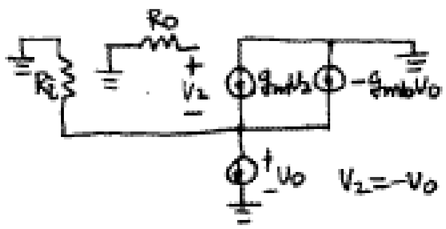
$\approx \frac{1}{(g_m + g_{mb})R_i}$

$= \frac{1}{(4.2\times 10^{-3} + 8.1\times 10^{-4})10^6}$
$= 2.00\times 10^{-4}$

$A = A_\infty \frac{R}{1+R} + \frac{d}{1+R}$
$= 1\frac{838}{1+838} + \frac{2.00\times 10^{-4}}{1+838}$
$= 0.999$

Problem 3 – Continued

$a_v = 0$



$$R_{in}(a_v=0) = R_i + \frac{1}{g_m} \| \frac{1}{g_{mb}} \approx R_i = 1M\Omega$$

$$\mathcal{R}(\text{short}) = \mathcal{R} = 838$$

$$\mathcal{R}(\text{open}) = 0 \quad (v_i = 0)$$

$$R_{in} = R_{in}(a_v=0)\frac{1+\mathcal{R}(\text{short})}{1+\mathcal{R}(\text{open})} \approx R_i \frac{1+\mathcal{R}}{1+0}$$

$$= R_i(1+\mathcal{R}) = 1M(1+838) = 839M\Omega$$

$$R_{out}(a_v=0) = R_i \| \frac{1}{g_m} \| \frac{1}{g_{mb}} \approx \frac{1}{g_m} \| \frac{1}{g_{mb}}$$

$$= \frac{1}{g_m + g_{mb}} = \frac{1}{4.2\times10^{-3} + 8.1\times10^{-4}} = 200\Omega$$

$$\mathcal{R}(\text{short}) = 0 \quad (v_o = 0)$$

$$\mathcal{R}(\text{open}) = \mathcal{R}$$

$$R_{out} = R_{out}(a_v=0)\frac{1+\mathcal{R}(\text{short})}{1+\mathcal{R}(\text{open})}$$

$$\approx \frac{1}{g_m+g_{mb}}\frac{1+0}{1+\mathcal{R}} = \frac{1}{g_m+g_{mb}}\frac{1}{1+\mathcal{R}}$$

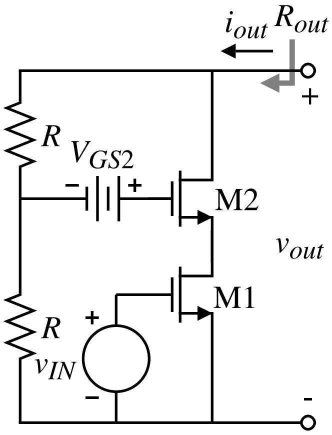
$$= 200\frac{1}{1+838} = 0.238\Omega$$

Problem 4 - (10 points)

Use the Blackman's formula (see below) to calculate the small-signal output resistance of the stacked MOSFET configuration having identical drain-source drops for both transistors. Express your answer in terms of all the pertinent small-signal parameters and then simplify your answer if $g_m > g_{ds} > (1/R)$. Assume the MOSFETs are identical.

$$R_{out} = R_{out}\,(g_m{=}0)\left[\frac{1 + RR(\text{output port shorted})}{1 + RR(\text{output port open})}\right]$$

(You may use small-signal analysis if you wish but this circuit seems to be one of the rare cases where feedback analysis is more efficient.)



S02E3P2

Solution

$$R_{out}\,(g_{m2}{=}0) = 2R\|(r_{ds1} + r_{ds2}) = \frac{2R(r_{ds1}+r_{ds2})}{2R+r_{ds1}+r_{ds2}}$$

RR(port shorted) = ?

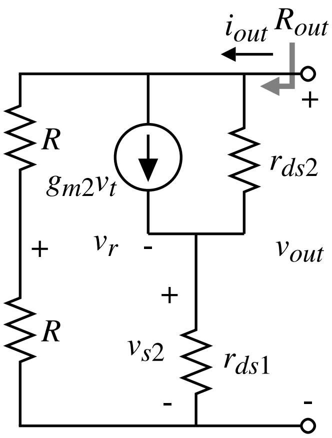
$$v_r = 0 - v_{s2} = -\,g_{m2}v_t\left(\frac{r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}}\right)$$

$$\Rightarrow \qquad RR(\text{port shorted}) = \frac{g_{m2}r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}}$$

RR(port open) = ?

$$v_r = -\,g_{m2}v_t\left(\frac{r_{ds2}(r_{ds1}+R)}{r_{ds1}+r_{ds2}+2R}\right)$$

$$\Rightarrow \qquad RR(\text{port open}) = \frac{g_{m2}r_{ds2}(r_{ds1}+R)}{r_{ds1}+r_{ds2}+2R}$$



S02E3S2

$$\therefore \qquad R_{out} = \frac{2R(r_{ds1}+r_{ds2})}{2R+r_{ds1}+r_{ds2}}\left[\frac{1 + \dfrac{g_{m2}r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}}}{1 + \dfrac{g_{m2}r_{ds2}(r_{ds1}+R)}{r_{ds1}+r_{ds2}+2R}}\right] = 2R\left(\frac{r_{ds1}+r_{ds2}+g_{m2}r_{ds1}r_{ds2}}{r_{ds1}+r_{ds2}+2R+g_{m2}r_{ds2}(r_{ds1}+R)}\right)$$

Using the assumptions of $g_m > g_{ds} > (1/R)$ we can simplify R_{out} as

$$R_{out} \approx 2R\left(\frac{g_{m2}r_{ds1}r_{ds2}}{g_{m2}r_{ds2}R}\right) = \underline{\underline{2r_{ds1}}}$$

What is the insight? Well there are two feedback loops, one a series (the normal cascode) and one a shunt. Apparently they are working against each other and the effective output resistance is pretty much what it would be if there were two transistors in series without any feedback.

Problem 5 – (10 points)

Find the dominant roots of the MOS follower and the BJT follower for the buffered, class-A op amp of Ex. 7.1-2. Use the capacitances of Table 3.2-1. Compare these root locations with the fact that GB = 5MHz? Assume the capacitances of the BJT are C_π = 10pf and C_μ = 1pF.

Solution

The model of just the output buffer of Ex. 7.1-2 is shown.

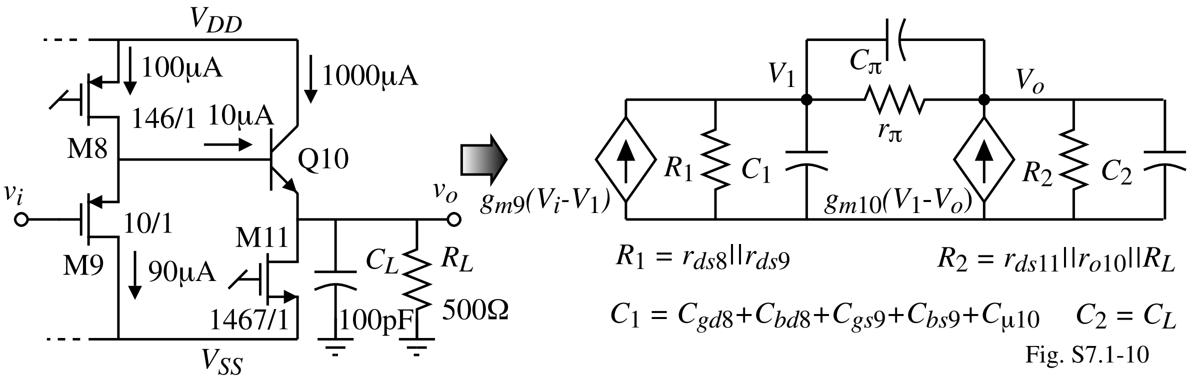


Fig. S7.1-10

The nodal equations can be written as,

$$g_{m9}V_i = (g_{m9} + G_1 + g_{\pi10} + sC_{\pi10} + sC_1)V_1 - (g_{\pi10} + sC_{\pi10})V_o$$

$$0 = -(g_{m10} + g_{\pi10} + sC_{\pi10})V_1 + (g_{m10} + G_2 + g_{\pi10} + sC_{\pi10} + sC_2)V_o$$

Solving for V_o/V_i gives,

$$\frac{V_o}{V_i} = \frac{g_{m9}(g_{m10}+ g_{\pi10}+ sC_{\pi10})}{(g_{\pi10} + sC_{\pi10})(g_{m9}+ G_1 + G_2 + sC_1 + sC_2) + (g_{m10}+ G_2+ sC_2)(g_{m9}+ G_1 + sC_1)}$$

$$\frac{V_o}{V_i} = \frac{g_{m9}(g_{m10}+ g_{\pi10}+ sC_{\pi10})}{a_0 + sa_1 + s^2a_2}$$

where

$$a_0 = g_{m9}g_{\pi10} + g_{\pi10}G_1 + g_{\pi10}G_2 + g_{m9}g_{m10} + g_{m10}G_1 + g_{m9}G_2 + G_1G_2$$

$$a_1 = g_{m9}C_{\pi10}+G_1C_{\pi10}+G_2C_{\pi10}+g_{\pi10}C_1+g_{\pi10}C_2+g_{m10}C_1+G_2C_1+g_{m9}C_2+G_1C_2$$

$$a_2 = C_{\pi10}C_1 + C_{\pi10}C_2 + C_1C_2$$

The numerical value of the small signal parameters are:

$$g_{m10} = \frac{1\text{mA}}{25.9\text{mV}} = 38.6\text{mS},\ G_2 = 2\text{mS},\ g_{\pi10} = 386\mu\text{S},\ g_{m9} = \sqrt{2\cdot50\cdot10\cdot90} = 300\mu\text{S},$$

$$G_1 = g_{ds8} + g_{ds9} = 0.05\cdot100\mu\text{A} + 0.05\cdot90\mu\text{A} = 9.5\mu\text{S}$$

$$C_2 = 100\text{pF},\ C_{\pi10} = 10\text{pF},\ C_1 = C_{gs9} + C_{bs9} + C_{bd8} + C_{gd8} + C_{\mu10}$$

$$C_{gs9} = C_{ov} +0.667C_{ox}W_9L_9 = (220\text{x}10^{-12})(10\text{x}10^{-6})+0.667(24.7\text{x}10^{-4})(10\text{x}10^{-12}) = 18.7\text{fF}$$

Problem 5 – Continued

$C_{bs9} = 560\text{x}10^{-6}(30\text{x}10^{-12})+350\text{x}10^{-12}(26\text{x}10^{-6}) = 25.9\text{fF}$

(Assumed area=3μmx10μm = 30μm and perimeter is 3μm+10μm+3μm+10μm = 26μm)

$C_{bd8} = 560\text{x}10^{-6}(438\text{x}10^{-12})+350\text{x}10^{-12}(298\text{x}10^{-6}) = 349\text{fF}$

$C_{gd8} = C_{ov} = (220\text{x}10^{-12})(146\text{x}10^{-6}) = 32.1\text{fF}$

$\therefore \quad C_1 = 18.7\text{fF} + 25.9\text{fF} + 349\text{fF} + 32.1\text{fF} + 1000\text{fF} = 1.43\text{pF}$

(We have ignored any reverse bias influence on pn junction capacitors.)

The dominant terms of a_0 , a_1, and a_2 based on these values are shown in boldface above.

$$\therefore \frac{V_o}{V_i} \approx \frac{g_{m9}(g_{m10}+ g_{\pi10}+ sC_{\pi10})}{g_{m9}g_{m10}+ g_{\pi10}G_2+s(G_2C_{\pi10}+g_{\pi10}C_2+g_{m10}C_1+g_{m9}C_2)+s^2C_2C_{\pi10}}$$

$$\frac{V_o}{V_i} = \frac{g_{m9}g_{m10}}{g_{m9}g_{m10}+ g_{\pi10}G_2}\left[\frac{1 + \frac{sC_{\pi10}}{g_{m10}}}{1 + s\left(\frac{G_2C_{\pi10}+g_{\pi10}C_2+g_{m10}C_1+g_{m9}C_2}{g_{m9}g_{m10}+ g_{\pi10}G_2}\right)+ s^2\frac{C_2C_{\pi10}}{g_{m9}g_{m10}+ g_{\pi10}G_2}}\right]$$

Assuming negative real axis roots widely spaced gives,

$$p_1 = -\frac{1}{\text{a}} = \frac{-(g_{m9}g_{m10}+ g_{\pi10}G_2)}{G_2C_{\pi10}+g_{\pi10}C_2+g_{m10}C_1+g_{m9}C_2} = -\frac{1.235\text{x}10^{-5}}{1.465\text{ x}10^{-13}} = \underline{\underline{-84.3\text{ x}10^6\text{ rads/sec.}}}$$

$= -13.4\text{MHz}$

$$p_2 = -\frac{\text{a}}{\text{b}} = \frac{-(G_2C_{\pi10}+g_{\pi10}C_2+g_{m10}C_1+g_{m9}C_2)}{C_2C_{\pi10}} = -\frac{1.465\text{ x}10^{-13}}{100\text{x}10^{-12}\cdot10\text{x}10^{-12}}$$

$= \underline{\underline{-146.5\text{ x}10^6\text{ rads/sec.}}} \rightarrow -23.32\text{MHz}$

$$z_1 = -\frac{g_{m10}}{C_{\pi10}} = -\frac{38.6\text{x}10^{-3}}{10\text{ x}10^{-12}} = \underline{\underline{-3.86\text{ x}10^9\text{ rads/sec.}}} \rightarrow -614\text{MHz}$$

We see that neither p_1 or p_2 is greater than $10GB$ if GB = 5MHz so they will deteriorate the phase margin of the amplifier of Ex. 7.1-2.