

Homework Assignment No. 13 - Solutions

Problem 1 - (10 points)

Problem 7.4-3 - Derive Eq. (17). If $A = 2$, at what value of v_{in}/nV_t will $i_{out} = 5I_5$ or $5I_b$ if $b=1$?

Solution

Start with the following relationships:

$$i_1 + i_2 = I_5 + A(i_2 - i_1) \quad \text{Eq. (15)}$$

and $\frac{i_2}{i_1} = \exp\left(\frac{v_{in}}{nV_t}\right)$ Eq. (16)

Defining $i_{out} = b(i_2 - i_1)$ solve for i_2 and I_1 .

$$i_1 + i_1 \exp\left(\frac{v_{in}}{nV_t}\right) = I_5 + Ai_1 \exp\left(\frac{v_{in}}{nV_t}\right) - Ai_1$$

or $i_1[(1+A) + (1-A) \exp\left(\frac{v_{in}}{nV_t}\right)] = I_5 \quad \rightarrow \quad i_1 = \frac{I_5}{(1+A) + (1-A) \exp\left(\frac{v_{in}}{nV_t}\right)}$

Similarly for i_2 ,

$$i_2 = \frac{I_5 \exp\left(\frac{v_{in}}{nV_t}\right)}{(1+A) + (1-A) \exp\left(\frac{v_{in}}{nV_t}\right)}$$

$$\therefore i_{out} = b(i_2 - I_1) = i_{out} = (i_2 - I_1) = \frac{I_5 \left(\exp\left(\frac{v_{in}}{nV_t}\right) - 1 \right)}{(1+A) + (1-A) \exp\left(\frac{v_{in}}{nV_t}\right)} \quad \text{Eq. (17)}$$

Setting $i_{out} = 5I_5$ and solving for $\frac{v_{in}}{nV_t}$ gives,

$$5[3 - \exp\left(\frac{v_{in}}{nV_t}\right)] = \exp\left(\frac{v_{in}}{nV_t}\right) - 1 \quad \rightarrow \quad 16 = 6 \exp\left(\frac{v_{in}}{nV_t}\right) \quad \rightarrow \quad \exp\left(\frac{v_{in}}{nV_t}\right) = 2.667$$

$$\therefore \frac{v_{in}}{nV_t} = \ln(2.667) = \underline{\underline{0.9808}}$$

Problem 2 - (10 points)

Problem 7.5-5. Find the equivalent rms noise voltage of the op amp designed in Example 6.5-2 over a bandwidth of 1Hz to 100kHz. Use the values for KF of Example 7.5-1.

Solution

The circuit for this amplifier is shown.

The W/L ratios in microns are:

$$S_1 = S_2 = 12/1$$

$$S_3 = S_4 = 16/1$$

$$S_5 = 7/1$$

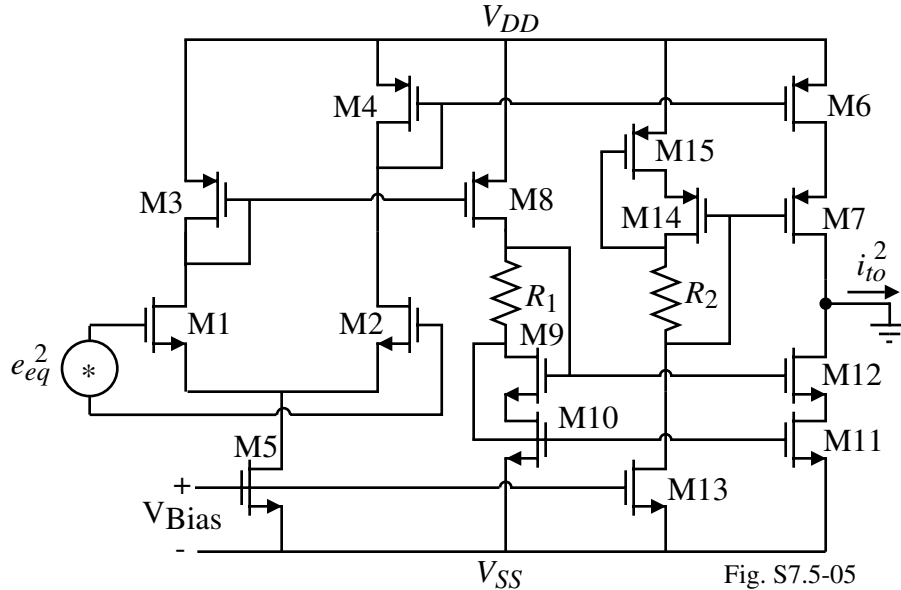
$$S_5 = 8.75/1$$

$$S_6 = S_7 = S_8 =$$

$$S_{14} = S_{15} = 40/1$$

$$S_9 = S_{10} = S_{11} =$$

$$S_{12} = 18.2/1$$



Find the short

circuit noise current at the output, i_{to}^2 , due to each noise-contributing transistor in the circuit (we will not include M7, M9, M12 and M14 because they are cascodes and their effective g_m is small. The result is,

$$i_{to}^2 = 2g_{m1}^2 e_{n1}^2 \left(\frac{g_{m8}^2}{g_{m3}^2} \right) + 2g_{m8}^2 e_{n3}^2 + 2g_{m8}^2 e_{n8}^2 + 2g_{m11}^2 e_{n10}^2$$

where we have assumed that $g_{m1}=g_{m2}$, $g_{m3}=g_{m4}$, $g_{m6}=g_{m8}$, and $g_{m10}=g_{m11}$ and

$e_{n1}=e_{n2}$, $e_{n3}=e_{n4}$, $e_{n6}=e_{n8}$, and $e_{n10}=e_{n11}$. Dividing i_{to}^2 by the transconductance gain gives

$$e_{eq}^2 = \frac{i_{to}^2}{g_{m1}^2 g_{m8}^2 / g_{m3}^2} = 2e_{n1}^2 + 2 \left(\frac{g_{m3}^2}{g_{m1}^2} \right) e_{n3}^2 + 2 \left(\frac{g_{m3}^2}{g_{m1}^2} \right) e_{n8}^2 + 2 \left(\frac{g_{m3}^2 g_{m11}^2}{g_{m1}^2 g_{m8}^2} \right) e_{n10}^2$$

The values of the various parameters are:

$$g_{m1} = 251 \mu\text{S}, g_{m3} = 282.5 \mu\text{S}, g_{m8} = 707 \mu\text{S}, \text{ and } g_{m11} = 707 \mu\text{S}.$$

$$\therefore e_{eq}^2 = 2e_{n1}^2 \left[1 + 1.266 \left(\frac{e_{n3}^2}{e_{n1}^2} + \frac{e_{n8}^2}{e_{n1}^2} + \frac{e_{n10}^2}{e_{n1}^2} \right) \right]$$

Problem 7.5-5 – Continued

1/f Noise:

Using the results of Ex. 7.5-1 we get $B_N = 7.36 \times 10^{-22} (\text{V}\cdot\text{m})^2$ and $B_P = 2.02 \times 10^{-22} (\text{V}\cdot\text{m})^2$

$$e_{n1}^2 = \frac{B_N}{fW_1L_1} = \frac{7.36 \times 10^{-22}}{f \cdot 12 \times 10^{-12}} = \frac{6.133 \times 10^{-11}}{f} \text{ V}^2/\text{Hz}$$

$$\frac{e_{n3}^2}{e_{n1}^2} = \frac{B_P \cdot f \cdot W_1L_1}{B_N \cdot f \cdot W_3L_3} = \frac{B_P \cdot W_1L_1}{B_N \cdot W_3L_3} = \frac{2.02 \cdot 12}{7.36 \cdot 16} = 0.2058$$

$$\frac{e_{n8}^2}{e_{n1}^2} = \frac{B_P \cdot f \cdot W_1L_1}{B_N \cdot f \cdot W_8L_3} = \frac{B_P \cdot W_1L_1}{B_N \cdot W_3L_3} = \frac{2.02 \cdot 12}{7.36 \cdot 40} = 0.0823$$

$$\frac{e_{n10}^2}{e_{n1}^2} = \frac{B_N \cdot f \cdot W_1L_1}{B_N \cdot f \cdot W_{10}L_{10}} = \frac{B_P \cdot W_1L_1}{B_N \cdot W_3L_3} = \frac{12}{18.2} = 0.6593$$

$$\therefore e_{eq}^2 = 2 \frac{6.133 \times 10^{-11}}{f} [1 + 1.266(0.2058 + 0.0823 + 0.6593)] = 2 \frac{6.133 \times 10^{-11}}{f} 2.1995$$

$$e_{eq}^2 = \frac{2.1995 \times 10^{-10}}{f} \text{ V}^2/\text{Hz}$$

Thermal noise:

$$e_{n1}^2 = \frac{8kT}{3g_{m1}} = \frac{8 \cdot 1.38 \times 10^{-23} \cdot 300}{3 \cdot 251 \times 10^{-6}} = 4.398 \times 10^{-17} \text{ V}^2/\text{Hz}$$

$$\frac{e_{n3}^2}{e_{n1}^2} = \frac{g_{m1}}{g_{m3}} = \frac{251}{282.4} = 0.8888 \quad \text{and} \quad \frac{e_{n8}^2}{e_{n1}^2} = \frac{e_{n10}^2}{e_{n1}^2} = \frac{g_{m1}}{g_{m8}} = \frac{251}{707} = 0.355$$

The corner frequency is $f_c = 2.698 \times 10^{-10} / 2.66 \times 10^{-16} = 1.01 \times 10^6$ Hz. Therefore in a 1Hz to 100kHz band, the noise is $1/f$. Solving for the *rms* value gives,

$$\begin{aligned} V_{eq}^2(\text{rms}) &= \int_1^{100,000} \frac{2.698 \times 10^{-10}}{f} df = 2.698 \times 10^{-10} [\ln(100,000) - \ln(1)] \\ &= 3.1062 \times 10^{-9} \text{ V}^2(\text{rms}) \end{aligned}$$

$$\therefore V_{eq}(\text{rms}) = \underline{\underline{55.73 \mu\text{V}(\text{rms})}}$$

Problem 3 - (10 points)

Problem 7.6-1 - If the W and L of all transistor in Fig. 7.6-3 are $100\mu\text{m}$ and $1\mu\text{m}$, respectively, find the lowest supply voltage that gives a zero value of $ICMR$ if the dc current in M5 is $100\mu\text{A}$.

Solution:

$$I_5 = 100 \mu\text{A} , \text{ and } \left(\frac{W}{L}\right) = 100$$

$$V_{IC}(\text{max}) = V_{DD} + V_{T1}(\text{min}) - V_{dsat3}$$

and, $V_{IC}(\text{min}) = V_{dsat1} + V_{T1}(\text{max}) + V_{dsat5}$

The input common-mode range is

$$ICMR = V_{IC}(\text{max}) - V_{IC}(\text{min})$$

For $ICMR=0$

$$V_{DD} = V_{dsat1} + V_{dsat5} + V_{dsat3} + V_{T1}(\text{max}) - V_{T1}(\text{min})$$

or,
$$V_{DD} = \sqrt{\frac{2I_1}{K_N S_1}} + \sqrt{\frac{2I_5}{K_N S_5}} + \sqrt{\frac{2I_3}{K_P S_3}} + V_{T1}(\text{max}) - V_{T1}(\text{min})$$

or,
$$V_{DD} = \underline{\underline{0.671\text{V}}}$$

Problem 4 - (10 points)

Problem 7.6-2 - Repeat Problem 7.6-1 if M1 and M2 are natural MOSFETs with a $V_T = 0.1\text{V}$ and the other MOSFET parameters are given in Table 3.1-2.

Solution

$$V_{T1} = 0.1 \text{ V}, I_5 = 100 \text{ } \mu\text{A}, \text{ and } \left(\frac{W}{L}\right) = 100$$

Let, the variation in the threshold voltage be $\pm 20\%$

$$\text{or, } \Delta V_{T1} = \pm 0.02 \text{ V}$$

$$V_{IC}(\text{max}) = V_{DD} + V_{T1}(\text{min}) - V_{dsat3}$$

$$\text{and, } V_{IC}(\text{min}) = V_{dsat1} + V_{T1}(\text{max}) + V_{dsat5}$$

The input common-mode range is

$$ICMR = V_{IC}(\text{max}) - V_{IC}(\text{min})$$

For $ICMR=0$

$$V_{DD} = V_{dsat1} + V_{dsat5} + V_{dsat3} + V_{T1}(\text{max}) - V_{T1}(\text{min})$$

$$\text{or, } V_{DD} = \sqrt{\frac{2I_1}{K'_N S_1}} + \sqrt{\frac{2I_5}{K'_N S_5}} + \sqrt{\frac{2I_3}{K'_P S_3}} + V_{T1}(\text{max}) - V_{T1}(\text{min})$$

$$\text{or, } V_{DD} = \underline{\underline{0.411\text{V}}}$$

Problem 5 - (10 points)

Problem 7.6-3 - Repeat Problem 7.6-1 if M1 and M2 are depletion MOSFETs with a $V_T = -1\text{V}$ and the other MOSFET parameters are given in Table 3.1-2.

Solution

$$V_{T1} = -1 \text{ V}, I_5 = 100 \text{ } \mu\text{A}, \text{ and } \left(\frac{W}{L}\right) = 100$$

Let, the variation in the threshold voltage be $\pm 20\%$

$$\text{or, } \Delta V_{T1} = \pm 0.2 \text{ V}$$

$$V_{IC}(\text{max}) = V_{DD} + V_{T1}(\text{min}) - V_{dsat3}$$

$$\text{and, } V_{IC}(\text{min}) = V_{dsat1} + V_{T1}(\text{max}) + V_{dsat5}$$

The input common-mode range is

$$ICMR = V_{IC}(\text{max}) - V_{IC}(\text{min})$$

For ICMR=0

$$V_{DD} = V_{dsat1} + V_{dsat5} + V_{dsat3} + V_{T1}(\text{max}) - V_{T1}(\text{min})$$

$$\text{or, } V_{DD} = \sqrt{\frac{2I_1}{K'_N S_1}} + \sqrt{\frac{2I_5}{K'_N S_5}} + \sqrt{\frac{2I_3}{K'_P S_3}} + V_{T1}(\text{max}) - V_{T1}(\text{min})$$

$$\text{or, } V_{DD} = \underline{\underline{0.771\text{V}}}$$